

1) Find the domain of the vector function:

a) $\mathbf{r}(t) = \left\langle \frac{1}{t+1}, \frac{t}{2}, -3t \right\rangle$

b) $\mathbf{r}(t) = \left\langle \sqrt{4-t^2}, t^2, -6t \right\rangle$

c) $\mathbf{r}(t) = \mathbf{F}(t) \times \mathbf{G}(t)$, where $\mathbf{F}(t) = \sin t \mathbf{i} + \cos t \mathbf{j}$ and $\mathbf{G}(t) = \sin t \mathbf{j} + \cos t \mathbf{k}$

d) $\mathbf{r}(t) = \mathbf{F}(t) \times \mathbf{G}(t)$, where $\mathbf{F}(t) = t^3 \mathbf{i} - t \mathbf{j} + t \mathbf{k}$ and $\mathbf{G}(t) = \sqrt[3]{t} \mathbf{i} + \frac{1}{t+1} \mathbf{j} + (t+2) \mathbf{k}$

2) Find the limit: (Use L'Hospital's Rule when needed.)

a) $\lim_{t \rightarrow 0^+} \langle \cos t, \sin t, t \ln t \rangle$

b) $\lim_{t \rightarrow 0} \left\langle \frac{e^t - 1}{t}, \frac{\sqrt{1+t} - 1}{t}, \frac{3}{1+t} \right\rangle$

c) $\lim_{t \rightarrow \infty} \left(\tan^{-1} t \mathbf{i} + e^{-2t} \mathbf{j} + \frac{\ln t}{t} \mathbf{k} \right)$

d) $\lim_{t \rightarrow \infty} \left(e^{-t} \mathbf{i} + \frac{1}{t} \mathbf{j} + \frac{t}{t^2 + 1} \mathbf{k} \right)$

3) Evaluate (if possible) the vector function $\mathbf{r}(t) = \left\langle \ln t, \frac{1}{t}, 3t \right\rangle$ at each given value of t .

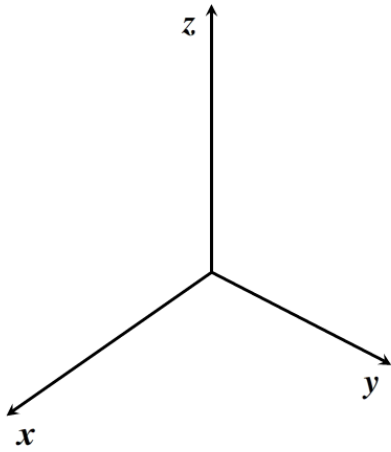
- a) $\mathbf{r}(2)$
- b) $\mathbf{r}(-3)$
- c) $\mathbf{r}(t-4)$
- d) $\mathbf{r}(1+\Delta t) - \mathbf{r}(1)$

4) Find $\|\mathbf{r}(t)\|$ if $\mathbf{r}(t) = \langle \sqrt{t}, 3t, -4t \rangle$.

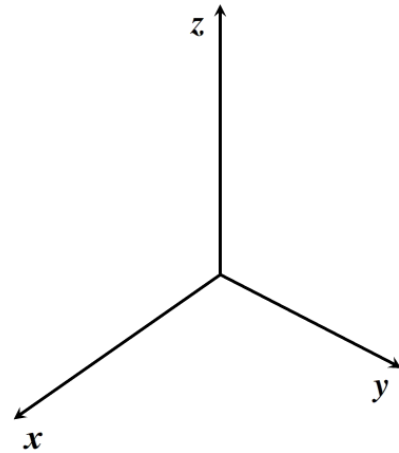
5) Represent the line segment from $P(0, 2, -1)$ to $Q(4, 7, 2)$ by a vector function and by a set of parametric equations.

- 6) Sketch the curve with the given vector function. Indicate with an arrow the direction in which t increases.

$$\mathbf{r}(t) = \langle t^2, t^4, t^6 \rangle$$



$$\mathbf{r}(t) = \langle t, \cos 2t, \sin 2t \rangle$$



- 7) Show that the curve with parametric equation $x = t \cos t$, $y = t \sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$.

- 8) Find a vector function that represents the curve of intersection of the two surfaces: the cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

9) Find a vector function that represents the curve of intersection of the two surfaces: the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

10) Is the vector function $\mathbf{r}(t) = \begin{cases} \mathbf{i} + \mathbf{j} & t \geq 2 \\ -\mathbf{i} + \mathbf{j} & t < 2 \end{cases}$ continuous at $t = 2$?

11) Two particles travel along the space curves $\mathbf{r}(t) = \langle t^2, 7t - 12, t^2 \rangle$ and $\mathbf{u}(t) = \langle 4t - 3, t^2, 5t - 6 \rangle$. A collision will occur at the point of intersection if both particles are at the point of intersection at the same time.

- At what times do the particles paths intersect?
- At what time and point do the particles collide?